18

Birzeit University-Mathematics Department
Calculus II-Math 132

First Exam — Second Semester 2013/2014

Name(Arabic):..................... Number:.............

Instructor of Discussion(Arabic):.............. Section:.............

Question 1.(19%) Circle the correct answer:

12

(1) Let $y = x^{\tan^{-1} x}$, then $y' =$

(a) $\tan^{-1} x \ln x$.

(b) $x^{\tan^{-1} x}\left(\frac{\ln x}{1+x^2} + \frac{\tan^{-1} x}{x}\right)$.

(c) $x^{\tan^{-1} x}\left((1+x^2)\ln x + (\tan^{-1} x)\ln x\right)$.

(d) $x^{\tan^{-1} x}\left(\frac{x}{1+x^2} + \tan^{-1} x\right)$.

$\ln y = \ln x^{\tan^{-1}x}$

$\ln y = \tan^{-1}x \ln x$

$\frac{1}{y}\frac{dy}{dx} = \tan^{-1}x \frac{1}{x} + \ln x \frac{1}{1+x^2}$

$y' = \left(\frac{\tan^{-1}x}{x} + \frac{\ln x}{1+x^2}\right) x^{\tan^{-1}x}$

(2) $\int_2^4 \frac{dx}{x\sqrt{x^2-4}} =$

(a) $\frac{\pi}{4}$.

(b) $\frac{\pi}{2}$.

(c) $\frac{\pi}{6}$.

(d) $\frac{\pi}{3}$.

$\frac{1}{2} \sec^{-1} \frac{x}{2}$

$x^2 - 4 = u$

$du = 2x\,dx$

(3) $\int_2^4 \text{sech}(\ln x)dx =$

(a) $\tanh(4) - \tanh(2)$.

(b) $\ln 4 - \ln 2$.

(c) $\ln 17 - \ln 4$.

(d) $\ln 17 - \ln 5$.

$\ln x = u$

$du = \frac{1}{x}dx$

$x = e^u$

$\int e^u \text{sech}\, u\, du$

(4) $\lim_{x\to\infty}(x - \ln x) =$

(a) 0.

(b) ∞.

(c) 1.

(d) Does not exist.

(5) $\int_0^1 \frac{dx}{\sqrt{3+2x-x^2}} =$

(a) $\frac{\pi}{3}$.

(b) $\frac{\pi}{6}$.

(c) $-\frac{\pi}{6}$.

(d) $\frac{\pi}{4}$.

(6) $\int_1^e \frac{2^{\ln x}}{x} dx =$

(a) 1.

(b) $\frac{1}{\ln 2}$.

(c) $-\ln 2$.

(d) $\frac{2}{\ln 2}$.

(7) $4^{\log_2(4)} =$

(a) 2.

(b) 4.

(c) 8.

(d) 16.

$\Rightarrow 2^{\log_2 4^2} = 4^2 = 16$

(8) Let $f(x) = 2x + e^x$ then $(f^{-1})'(1) =$

(a) 1.

(b) $\frac{1}{2}$.

(c) $\frac{1}{3}$.

(d) $\frac{1}{2+e}$.

(9) $\int_0^{\pi/3} (\sec\theta)^4 d\theta =$

(a) 3.

(b) $\sqrt{3}$.

(c) $3\sqrt{3}$.

(d) $2\sqrt{3}$.

$\sec^2\theta \, (1+\tan^2\theta)\, d\theta$

$\int (1+u^2)\, du$

$u + \frac{u^3}{3} = \tan\theta + \frac{\tan^3\theta}{3}$

(10) The half-life of a radioactive element is 3500 years. The decay rate of the element is

$k = \quad t = \frac{\ln 2}{k}$

$k = \frac{\ln 2}{t} = \frac{\ln 2}{3500}$

(a) $3500 \ln 2$.

(b) $\frac{3500}{\ln 2}$.

(c) $\frac{\ln 2}{3500}$.

(d) $\ln 2$.

(11) $\int_0^{\pi/3} \cos^3 x dx =$

$\cos x (1 - \sin^2 x) dx$

$\sin x = u$

$du = \cos x \, dx$

$\int (1 - u^2) du$

$u - \frac{u^3}{3}$

$\sin x - \frac{\sin^3 x}{3} \Big|_0^{\frac{\pi}{3}}$

$\frac{\sqrt{3}}{2} - \frac{\sqrt{3}^3}{8} - 0$

$\frac{\sqrt{3}}{2} - \frac{\cancel{3}\sqrt{3}}{8 \cdot \cancel{3}}$

$\frac{4\sqrt{3}}{8} - \frac{\sqrt{3}}{8} = \frac{3\sqrt{3}}{8}$

(a) $3\sqrt{3}$.

(b) $\frac{\sqrt{3}}{8}$.

(c) $\frac{3\sqrt{3}}{8}$.

(d) $\frac{\sqrt{3}}{2}$.

(12) $\cos^{-1}(\cos(-\frac{\pi}{2})) =$

$0 \leq \cos^{-1} \leq \pi$

(a) $-\frac{\pi}{2}$.

(b) 0.

(c) $\frac{\pi}{2}$.

(d) None.

(13) The following functions grow from slowest to fastest

(a) $2^x, x^{10}, x^x, (\ln x)^x$.

(b) $(\ln x)^x, x^x, 2^x, x^{10}$.

(c) $x^{10}, 2^x, (\ln x)^x, x^x$.

(d) $x^{10}, 2^x, x^x, (\ln x)^x$.

$\sin^{-1} \quad -\frac{\pi}{2}, \frac{\pi}{2}$

(14) The range of the function $\cos(\sin^{-1} x)$ is

(a) $[0, 1]$.

(b) $[-1, 1]$.

(c) $[-1, 0]$.

(d) $[0, \pi]$.

(15) Let $f(x) = \frac{x}{1-x}$, $x \neq 1$ then $f^{-1}(x) =$

(a) $\frac{x}{1-x}$.

(b) $\frac{1-x}{x}$.

(c) $\frac{x}{x+1}$.

(d) $\frac{x+1}{x}$.

$y = \frac{x}{1+x}$

$y - yx = x$

$y = x + yx$

$x(1+y) = y$

(16) $\sin(\tan^{-1} x) =$

(a) $\frac{x}{\sqrt{1+x^2}}$.

(b) $\frac{1}{\sqrt{1+x^2}}$.

(c) $\frac{1}{\sqrt{1-x^2}}$.

(d) $\frac{x}{\sqrt{1-x^2}}$.

$u =$ $\sin(\cos^{-1}(x)) =$

$x(1+y) = y$

$x = \frac{y}{1+y}$

$y = \frac{x}{1+x}$

(17) Let $y = \log_2(\ln x^2)$ then $y' =$

(a) $\frac{1}{x \ln x}$.

(b) $\frac{1}{(\ln 2) x \ln x}$.

(c) $\frac{2}{(\ln 2) x \ln x}$.

(d) $\frac{1}{x^2 \ln(x^2)}$.

$\frac{\ln(\ln x^2)}{\ln 2}$

$2 \frac{\ln(\ln x)}{\ln 2}$

$\frac{2}{\ln 2} \frac{1}{x \ln x}$

(18) $\lim\limits_{x \to 0^+} \left(\frac{1}{x}\right)^x =$

(a) 1.

(b) 0.

(c) ∞.

(d) Does not exist.

$\ln\left(\frac{1}{x}\right)^x$

$x \ln \frac{1}{x}$

$\frac{\ln \frac{1}{x}}{\frac{1}{x}} = \frac{0}{0}$

(19) $\int_0^1 x \tan^{-1} x dx =$

(a) $\pi - 1$

(b) $\frac{\pi}{4} - 1$.

(c) $\frac{\pi}{4} - \frac{1}{2}$.

(d) $\frac{\pi}{2} - 1$.

$u = \tan^{-1} x$ $du = \frac{1}{1+x^2} dx$

$dv = x dx$ $v = \frac{x^2}{2}$

$\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx$

$1 + x^2 = u$

$du = 2x dx$

$\frac{(e^x - e^{-x})}{(e^x + e^{-x})}$

Question 2(6%) Use a substitution then integrate by parts to solve the integral

$$\int \sin(\ln x)dx$$

$z = \ln x \Rightarrow x = e^z$

$dz = \frac{1}{x} dx$

$\int e^z \sin z \, dz$

$u = e^z \quad du = e^z dz$

$dv = \sin z dz \quad v = -\cos z$

$-e^z \cos z + \int e^z \cos z \, dz$

$w = e^z \quad dw = e^z dz$

$ds = \cos z dz \quad s = \sin z$

$e^z \sin z - \int \sin z \, e^z dz$

$\int e^z \sin z \, dz = -e^z \cos z + e^z \sin z - \int \sin z \, e^z dz$

6

$\int e^z \sin z \, dz = \frac{1}{2}\left[-e^z \cos z + e^z \sin z\right]$

$= \frac{1}{2}\left[-e^{\ln x} \cos \ln x + e^{\ln x} \sin \ln x\right]$

5

$= \frac{1}{2}\left[-x \cos \ln x + x \sin \ln x\right]$

$\frac{x}{2}\left[\sin \ln x - \cos \ln x\right] + C$